Electrical Engineering

Mr. Abir Naqvi

Table of Content

Preliminary Remarks

The present study text "Electrical Engineering" is slightly different from other study texts of the B. Ed. in Technical Education programs.

Firstly, we have organised the chapters of this study text according to the outline of the entire specialised area "Electrical Engineering", which includes the content both for face-to-face and for self-learning phases. For a better understanding, you find the number of the chapter of the outline within brackets at the end of each chapter heading of this text.

Secondly we allocated the learning objectives to the respective chapters. We did this because of the large and comprehensive catalogue of learning objectives in order to facilitate the allocation of specific learning objectives to the referring chapter.

We hope to have simplified the understanding of the text by these measures.

1 Kirchhoff's Laws (Chapter 2)

Learning Objectives

After going through this chapter you will be able to

- know the difference between Kirchhoff's voltage law and current law
- apply *Kirchhoff's laws* to simple electric circuits and derive the basic circuit equations
- know the relevance of Wheatstone bridge
- have a deeper understanding of Ohm's law
- know how to relate the current voltage and resistance to each other

Preface

In all of the circuits examined during the face-to-face phase, Ohm's Law described the relationship between current, voltage, and resistance. These circuits have been relatively simple in nature. Many circuits are extremely complex and cannot be solved with Ohm's Law. These circuits have many power sources and branches which would make the use of Ohm's Law impractical or impossible.

Through experimentation in 1857 the German physicist Gustav Kirchhoff developed methods to solve complex circuits. Kirchhoff developed two conclusions, known today as Kirchhoff's Laws.

1.1 Law 1: Kirchhoff's Voltage Law

Kirchhoff's first law is also known as his "voltage law". The voltage law gives the relationship between the "voltage drops" around any closed loop in a circuit, and the voltage sources in that loop. The total of these two quantities is always equal. In equation form:

$$E_{source} = E_1 + E_2 + E_3 + \text{etc.} = I_1R_1 + I_2R_2 + I_3R_3 + \text{etc.}$$

$$\Sigma E_{source} = \Sigma IR \qquad (2\text{-}14)$$

Where the symbol Σ (the Greek letter sigma) means "the sum of."

Kirchhoff's voltage law can be applied only to closed loops (Figure 1). A closed loop must meet two conditions:

1) It must have one or more voltage sources.
2) It must have a complete path for current flow from any point, around the loop, and back to that point.

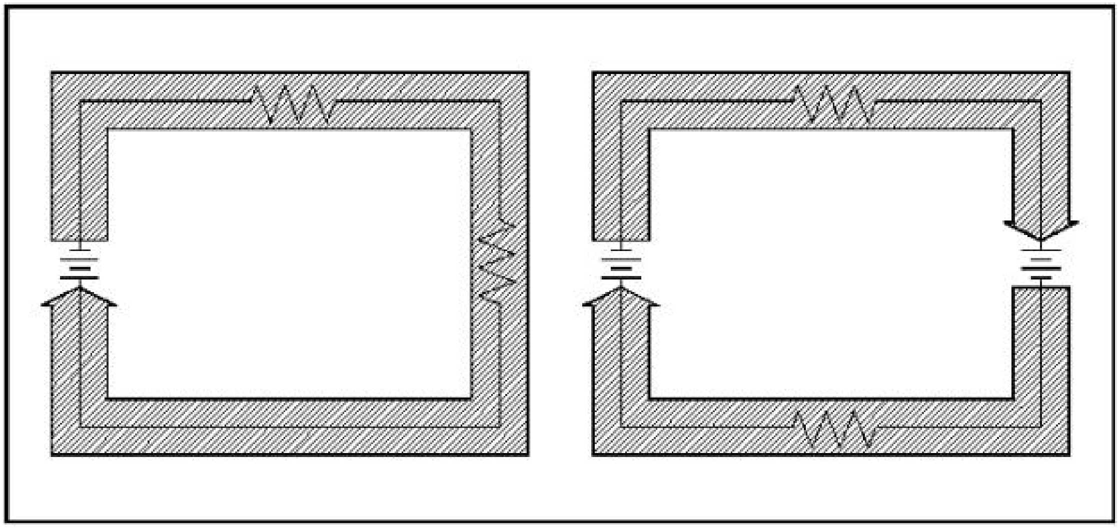


Figure 1 "Closed Loop"

You will remember that in a simple series circuit, the sum of the voltage drops around the circuit is equal to the applied voltage. Actually, this is Kirchhoff's voltage law applied to the simplest case, that is, where there is only one loop and one voltage source.

1.2 Law 2

The current arriving at any junction point in a circuit is equal to the current leaving that junction (Kirchhoff's Current Law).

Kirchhoff's two laws may seem obvious based on what we already know about circuit theory. Even though they may seem very simple, they are powerful tools in solving complex and difficult circuits.

Kirchhoff's laws can be related to conservation of energy and charge if we look at a circuit with one load and source. Since all of the power provided from the source is consumed by the load, energy and charge are conserved. Since voltage and current can be related to energy and charge, then Kirchhoff's laws are only restating the laws governing energy and charge conservation.

The mathematics involved becomes more difficult as the circuits become more complex. Therefore, the discussion here will be limited to solving only relatively simple circuits.

1.3 Wheatstone Bridge

1.3.1 General Relevance

For measuring accurately any electrical resistance **Wheatstone bridge** is widely used. There are two known resistors, one variable resistor and one unknown resistor connected in bridge form as shown below. By adjusting the variable resistor the current through the Galvanometer is made zero. When the electric current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance. In this way the value of unknown electrical resistance can easily be measured by using a **Wheatstone Bridge**.

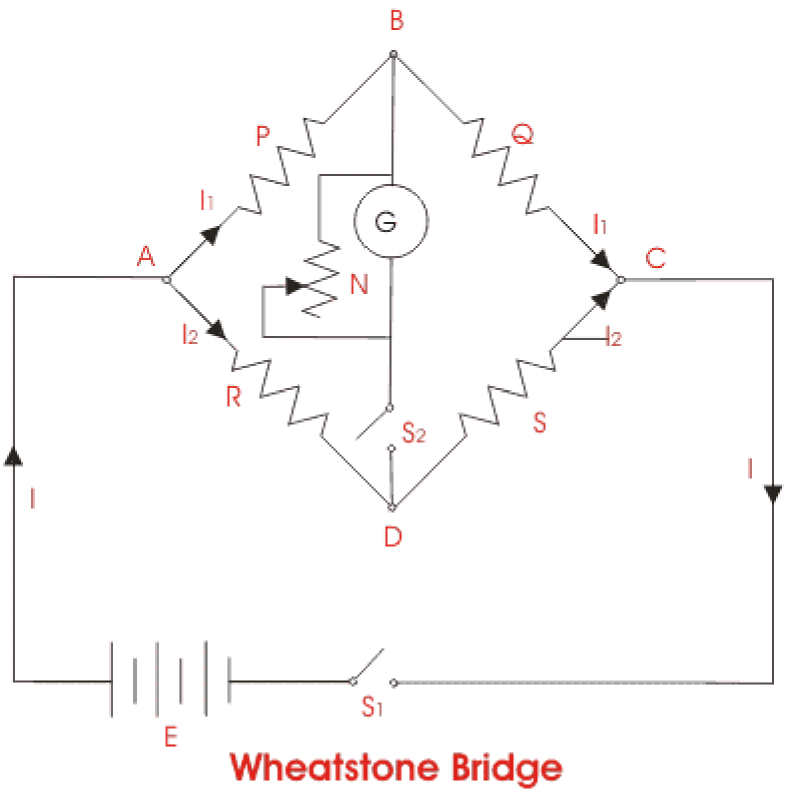


Figure 2 "Wheatstone Bridge"

1.3.2 Wheatstone Bridge Theory

The general arrangement of **Wheatstone bridge circuit** is shown in Figure 2. It is a four arms bridge circuit where arm AB, BC, CD and AD are consisting of electrical resistances P, Q, S and R respectively. Among these resistances P and Q are known fixed electrical resistances and these two arms are referred as ratio arms. An accurate and sensitive Galvanometer is connected between the terminals B and D through a switch S_2. The voltage source of this Wheatstone bridge is connected to the terminals A and C via a switch S_1 as shown. A variable resistor S is connected between point C and D. The potential at point D can be varied by adjusting the value of variable resistor. Suppose current I_1 and current I_2 are flowing through the paths ABC and ADC respectively. If we vary the electrical resistance value of arm CD the value of current I_2will also be varied as the voltage across A and C is fixed. If we continue to adjust the variable resistance one situation may comes when voltage drop across the resistor S that is $I_2.S$ is becomes exactly equal to voltage drop across resistor Q that is $I_1.Q$. Thus the potential at point B becomes equal to the potential at point D hence potential difference between these two points is zero hence current through galvanometer is nil. Then the deflection in the galvanometer is nil when the switch S_2 is closed.

1.4 Reading a Potentiometer (analog input)

A potentiometer is a simple knob that provides a variable resistance, which we can read into the Adriano board as an analog value. In this example, that value controls the rate at which an LED blinks.

We connect three wires to the Arduino board. The first goes to ground from one of the outer pins of the potentiometer. The second goes from 5 volts to the other outer pin of the potentiometer. The third goes from analog input 2 to the middle pin of the potentiometer.

By turning the shaft of the potentiometer, we change the amount of resistance on either side of the wiper which is connected to the centre pin of the potentiometer. This changes the relative "closeness" of that pin to 5 volts and ground, giving us a different analog input. When the shaft is turned all the way in one direction, there are 0 volts going to the pin, and we read 0. When the shaft is turned all the way in the other direction, there are 5 volts going to the pin and we read 1023. In between, analog Read() returns a number between 0 and 1023 that is proportional to the amount of voltage being applied to the pin.

1.4.1 OHM's Law

Ohm's Law deals with the relationship between voltage and current in an ideal conductor. This relationship states that:

The potential difference (voltage) across an ideal conductor is proportional to the current through it.

The constant of proportionality is called the "resistance", **R**.

Ohm's Law is given by:

$$\mathbf{V = I\,R}$$

where V is the potential difference between two points which include a **resistance** R. I is the current flowing through the resistance. For biological work, it is often preferable to use the **conductance**, g = 1/R; In this form Ohm's Law is:

$$\mathbf{I = g\,V}$$

1) Material that obeys Ohm's Law is called "**ohmic**" or "**linear**" because the potential difference across it varies linearly with the current.
2) Ohm's Law can be used to solve simple circuits. A complete circuit is one which is a closed loop. It contains at least one source of voltage (thus providing an increase of potential energy), and at least one potential drop i.e., a place where potential energy decreases. The sum of the voltages around a complete circuit is zero.
3) An increase of potential energy in a circuit causes a charge to move from a lower to a higher potential (i.e. voltage). Note the difference between potential energy and potential.

   Because of the electrostatic force, which tries to move a positive charge from a higher to a lower potential, there must be another 'force' to move charge from a lower potential to a higher inside the battery. This so-called force is called the **electromotive force**, or **emf**. The SI unit for the emf is a volt (and thus this is not really a force, despite its name). We will use a script E, the symbol $\mathcal{E}$, to represent the emf.

   A decrease of potential energy can occur by various means. For example, heat lost in a circuit due to some electrical resistance could be one source of energy drop.

   Because energy is conserved, the potential difference across an emf must be equal to the potential difference across the rest of the circuit. That is, Ohm's Law will be satisfied:

   $$\mathcal{E}\ \mathbf{= I\,R}$$
4) Here is a nice simulated experiment on Ohm's Law for you to test your understanding of this concept. Use the "back" button to return to this place.

1.4.2 How to relate the current voltage and resistance to each other

An electric circuit is formed when a conductive path is created to allow free electrons to continuously move. This continuous movement of free electrons through the conductors of a circuit is called a *current*, and it is often referred to in terms of "flow," just like the flow of a liquid through a hollow pipe.

The force motivating electrons to "flow" in a circuit is called *voltage*. Voltage is a specific measure of potential energy that is always relative between two points. When we speak of a certain amount of voltage being present in a circuit, we are referring to the measurement of how much *potential* energy exists to move electrons from one particular point in that circuit to another particular point. Without reference to *two* particular points, the term "voltage" has no meaning.

Free electrons tend to move through conductors with some degree of friction, or opposition to motion. This opposition to motion is more properly called *resistance*. The amount of current in a circuit depends on the amount of voltage available to motivate the electrons, and also the amount of resistance in the circuit to oppose electron flow. Just like voltage, resistance is a quantity relative between two points. For this reason, the quantities of voltage and resistance are often stated as being "between" or "across" two points in a circuit.

To be able to make meaningful statements about these quantities in circuits, we need to be able to describe their quantities in the same way that we might quantify mass, temperature, volume, length, or any other kind of physical quantity. For mass we might use the units of "kilogram" or "gram." For temperature we might use degrees Fahrenheit or degrees Celsius. Here are the standard units of measurement for electrical current, voltage, and resistance:

Quantity	Symbol	Unit of Measurement	Unit Abbreviation
Current	I	Ampere ("Amp")	A
Voltage	E *or* V	Volt	V
Resistance	R	Ohm	Ω

The "symbol" given for each quantity is the standard alphabetical letter used to represent that quantity in an algebraic equation. Standardized letters like these are common in the disciplines of physics and engineering, and are internationally recognized. The "unit abbreviation" for each quantity represents the alphabetical symbol used as a shorthand notation for its particular unit of measurement. And, yes, that strange-looking "horseshoe" symbol is the capital Greek letter Ω, just a character in a *foreign* alphabet (apologies to any Greek readers here).

Each unit of measurement is named after a famous experimenter in electricity: The *amp* after the Frenchman Andre M. Ampere, the *volt* after the Italian Alessandro Volta, and the *ohm* after the German Georg Simon Ohm.

The mathematical symbol for each quantity is meaningful as well. The "R" for resistance and the "V" for voltage are both self-explanatory, whereas "I" for current seems a bit weird. The "I" is thought to have been meant to represent "Intensity" (of electron flow), and the other symbol for voltage, "E," stands for "Electromotive force." From what research I've been able to do, there seems to be some dispute over the meaning of "I." The symbols "E" and "V" are interchangeable for the most part, although some texts reserve "E" to represent voltage across a source (such as a battery or generator) and "V" to represent voltage across anything else.

All of these symbols are expressed using capital letters, except in cases where a quantity (especially voltage or current) is described in terms of a brief period of time (called an "instantaneous" value). For example, the voltage of a battery, which is stable over a long period of time, will be symbolized with a capital letter "E," while the voltage peak of a lightning strike at the very instant it hits a power line would most likely be symbolized with a lower-case letter "e" (or lower-case "v") to designate that value as being at a single moment in time. This same lower-case convention holds true for current as well, the lower-case letter "i" representing current at some instant in time. Most direct-current (DC) measurements, however, being stable over time, will be symbolized with capital letters.

One foundational unit of electrical measurement, often taught in the beginnings of electronics courses but used infrequently afterwards, is the unit of the *coulomb*, which is a measure of electric charge proportional to the number of electrons in an imbalanced state. One coulomb of charge is equal to 6,250,000,000,000,000,000 electrons. The symbol for electric charge quantity is the capital letter "Q," with the unit of coulombs abbreviated by the capital letter "C." It so happens that the unit for electron flow, the amp, is equal to 1 coulomb of electrons passing by a given point in a circuit in 1 second of time. Cast in these terms, current is the *rate of electric charge motion* through a conductor.

As stated before, voltage is the measure of *potential energy per unit charge* available to motivate electrons from one point to another. Before we can precisely define what a "volt" is, we must understand how to measure this quantity we call "potential energy." The general metric unit for energy of any kind is the *joule*, equal to the amount of work performed by a force of 1 newton exerted through a motion of 1 meter (in the same direction). In British units, this is slightly less than 3/4 pound of force exerted over a distance of 1 foot. Put in common terms, it takes about 1 joule of energy to lift a 3/4 pound weight 1 foot off the ground, or to drag something a distance of 1 foot using a parallel pulling force of 3/4 pound. Defined in these scientific terms, 1 volt is equal to 1 joule of electric potential energy per (divided by) 1 coulomb of charge. Thus, a 9 volt battery releases 9 joules of energy for every coulomb of electrons moved through a circuit.

These units and symbols for electrical quantities will become very important to know as we begin to explore the relationships between them in circuits. The first, and perhaps most important, relationship between current, voltage, and resistance is called Ohm's Law, discovered by Georg Simon Ohm and published in his 1827 paper, *The Galvanic Circuit Investigated Mathematically*. Ohm's principal discovery was that the amount of electric current through a metal conductor in a circuit is directly proportional to the voltage impressed across it, for any given temperature. Ohm expressed his discovery in the form of a simple equation, describing how voltage, current, and resistance interrelate:

$$E = I R$$

In this algebraic expression, voltage (E) is equal to current (I) multiplied by resistance (R). Using algebra techniques, we can manipulate this equation into two variations, solving for I and for R, respectively:

$$I = \frac{E}{R} \qquad R = \frac{E}{I}$$

Let's see how these equations might work to help us analyze simple circuits:

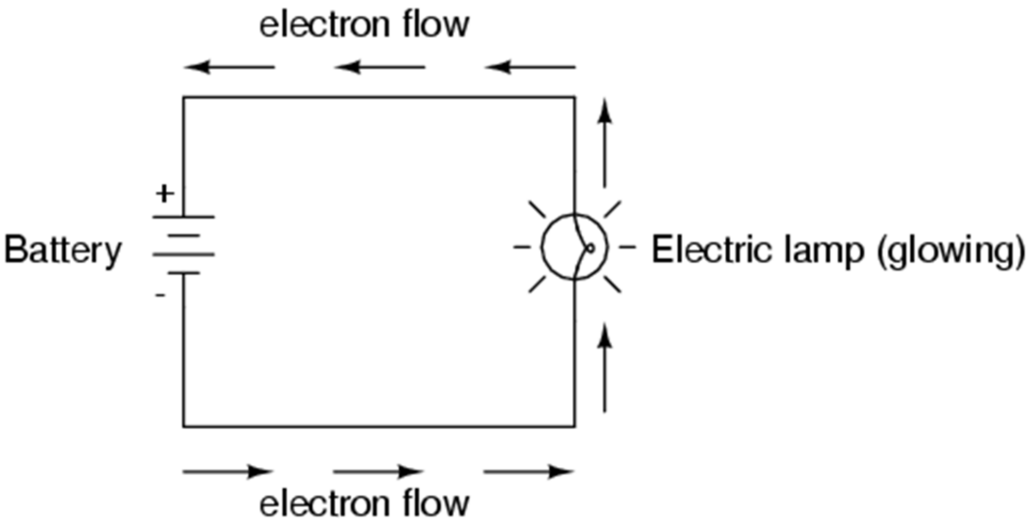
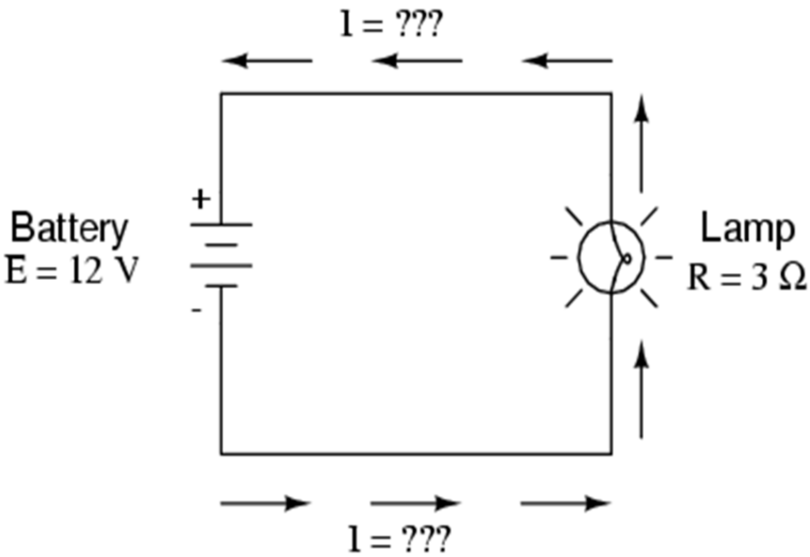


Figure 3 Circuit

In the circuit in Figure 3, there is only one source of voltage (the battery, on the left) and only one source of resistance to current (the lamp, on the right). This makes it very easy to apply Ohm's Law. If we know the values of any two of the three quantities (voltage, current, and resistance) in this circuit, you can use Ohm's Law to determine the third.

Activity 1

In this first example, we will calculate the amount of current (I) in a circuit, given values of voltage (E) and resistance (R):

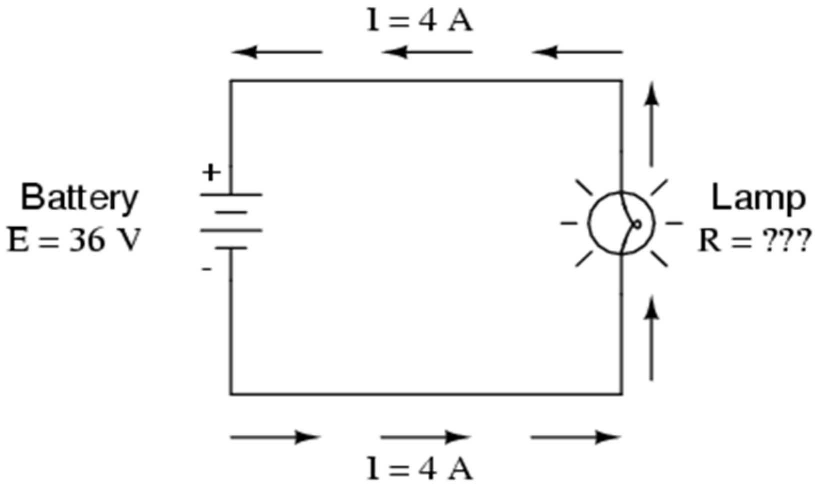


What is the amount of current (I) in this circuit?

$$I = \frac{E}{R} = \frac{12\ V}{3\ \Omega} = 4\ A$$

Activity 2

In this second example, calculate the amount of resistance (R) in a circuit, given values of voltage (E) and current (I):

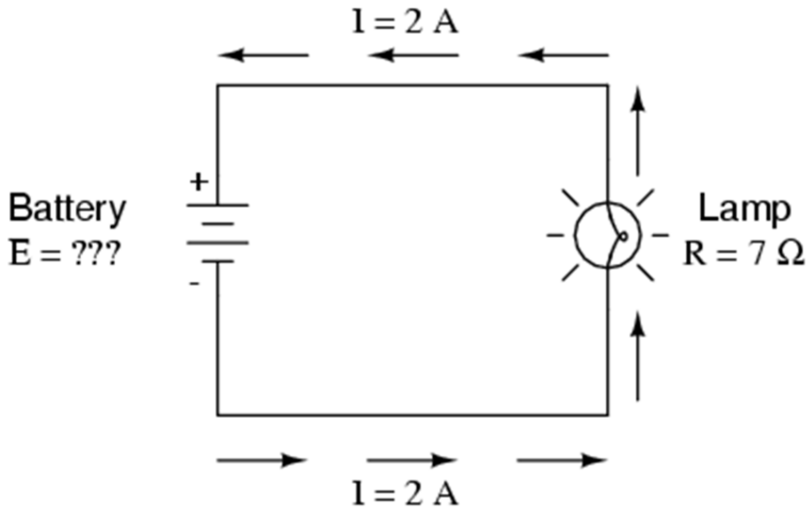


What is the amount of resistance (R) offered by the lamp?

$$R = \frac{E}{I} = \frac{36\ V}{4\ A} = 9\ \Omega$$

Activity 3

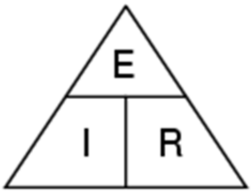
In the last example, calculate the amount of voltage supplied by a battery, given values of current (I) and resistance (R):



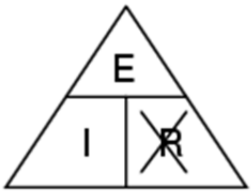
What is the amount of voltage provided by the battery?

$$E = IR = (2\ A)(7\ \Omega) = 14\ V$$

Ohm's Law is a very simple and useful tool for analyzing electric circuits. It is used so often in the study of electricity and electronics that it needs to be committed to memory by the serious student. For those who are not yet comfortable with algebra, there's a trick to remembering how to solve for any one quantity, given the other two. First, arrange the letters E, I, and R in a triangle like this:

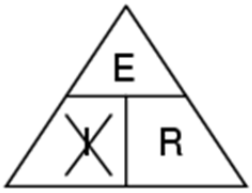


If you know E and I, and wish to determine R, just eliminate R from the picture and see what's left:



$$R = \frac{E}{I}$$

If you know E and R, and wish to determine I, eliminate I and see what's left:



$$I = \frac{E}{R}$$

Lastly, if you know I and R, and wish to determine E, eliminate E and see what's left:

$$E = I\,R$$

Eventually, you'll have to be familiar with algebra to seriously study electricity and electronics, but this tip can make your first calculations a little easier to remember. If you are comfortable with algebra, all you need to do is commit E=IR to memory and derive the other two formulae from that when you need them!

REVIEW:

1) Voltage measured in*volts*, symbolized by the letters "E" or "V".

2) Current measured in amps, symbolized by the letter "I".

3) Resistance measured in ohms, symbolized by the letter "R".

4)Ohm's Law: E = IR ; I = E/R ; R = E/I

2 Electromagnetism – Cathode Ray Oscilloscope (Chapter 3.5)

Objectives

After going through this chapter you will be able to

- know how to operate a cathode-ray oscilloscope
- know how to measure current with a multimeter
- know how to check a multimeter's internal fuse
- know how to select a proper meter range.

2.1 Introduction

The cathode-ray oscilloscope (CRO) is a common laboratory instrument that provides accurate time and amplitude measurements of voltage signals over a wide range of frequencies. Its reliability, stability, and ease of operation make it suitable as a general purpose laboratory instrument. The heart of the CRO is a cathode-ray tube shown schematically in Figure 4.

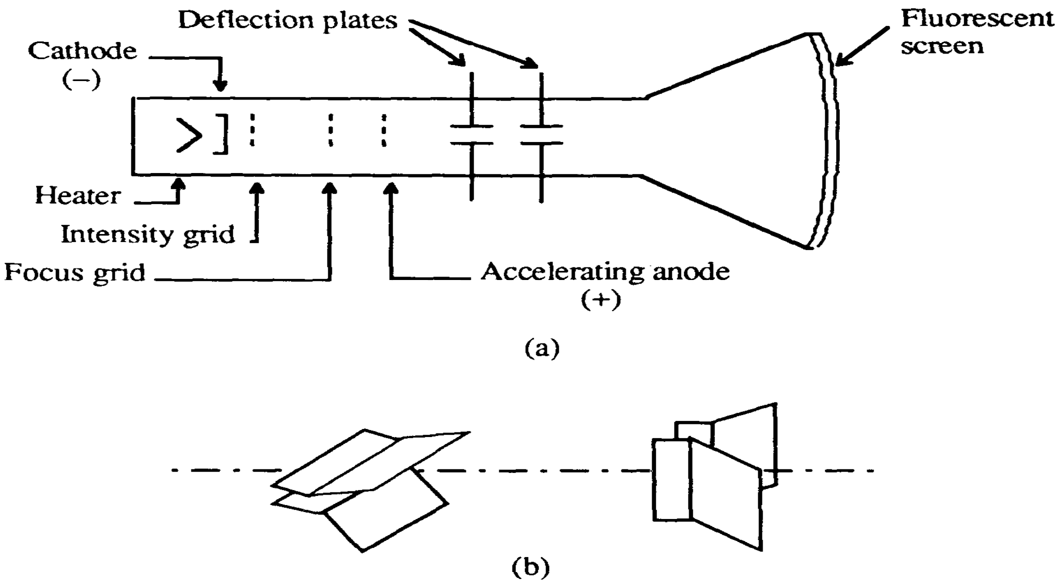


Figure 4 Cathode-ray tube: (a) schematic, (b) detail of the deflection plates

The cathode ray is a beam of electrons which are emitted by the heated cathode (negative electrode) and accelerated toward the fluorescent screen. The assembly of the cathode, intensity grid, focus grid, and accelerating anode (positive electrode) is called an *electron gun*. Its purpose is to generate the electron beam and control its intensity and focus. Between the electron gun and the fluorescent screen are two pairs of metal plates - one oriented to provide horizontal deflection of the beam and one pair oriented to give vertical deflection to the beam. These plates are thus referred to as the *horizontal* and *vertical deflection plates*. The combination of these two deflections allows the beam to reach any portion of the fluorescent screen.

Wherever the electron beam hits the screen, the phosphor is excited and light is emitted from that point. This conversion of electron energy into light allows us to write with points or lines of light on an otherwise darkened screen.

In the most common use of the oscilloscope the signal to be studied is first amplified and then applied to the vertical (deflection) plates to deflect the beam vertically and at the same time a voltage that increases linearly with time is applied to the horizontal (deflection) plates thus causing the beam to be deflected horizontally at a uniform (constant> rate. The signal applied to the vertical plates is thus displayed on the screen as a function of time. The horizontal axis serves as a uniform time scale.

The linear deflection or sweep of the beam horizontally is accomplished by use of a *sweep generator* that is incorporated in the oscilloscope circuitry. The voltage output of such a generator is that of a sawtooth wave as shown in Figure 5. Application of one cycle of this voltage difference, which increases linearly with time, to the horizontal plates causes the beam to be deflected linearly with time across the tube face. When the voltage suddenly falls to zero, as at points (a) (b) (c), etc...., the end of each sweep - the beam flies back to its initial position. The horizontal deflection of the beam is repeated periodically, the frequency of this periodicity is adjustable by external controls.

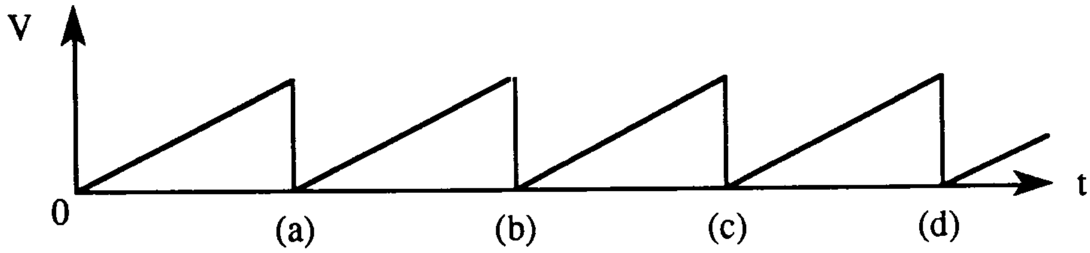


Figure 5 Voltage difference V between horizontal plates as a function of time t

To obtain steady traces on the tube face, an internal number of cycles of the unknown signal that is applied to the vertical plates must be associated with each cycle of the sweep generator. Thus, with such a matching of synchronization of the two deflections, the pattern on the tube face repeats itself and hence appears to remain stationary. The persistence of vision in the human eye and of the glow of the fluorescent screen aids in producing a stationary pattern. In addition, the electron beam is cut off (blanked) during flyback so that the retrace sweep is not observed.

2.1.1 CRO Operation

A simplified block diagram of a typical oscilloscope is shown in Figure 6. In general, the instrument is operated in the following manner. The signal to be displayed is amplified by the vertical amplifier and applied to the vertical deflection plates of the CRT. A portion of the signal in the vertical amplifier is applied to the **sweep trigger** as a triggering signal. The sweep trigger then generates a pulse coincident with a selected point in the cycle of the triggering signal. This pulse turns on the sweep generator, initiating the sawtooth wave form. The sawtooth wave is amplified by the horizontal amplifier and applied to the horizontal deflection plates. Usually, additional provisions signal are made for applying an external triggering signal or utilizing the 60 Hz line for triggering. Also the sweep generator may be bypassed and an external signal applied directly to the horizontal amplifier.

2.1.2 CRO Controls

The controls available on most oscilloscopes provide a wide range of operating conditions and thus make the instrument especially versatile. Since many of these controls are common to most oscilloscopes a brief description of them follows.

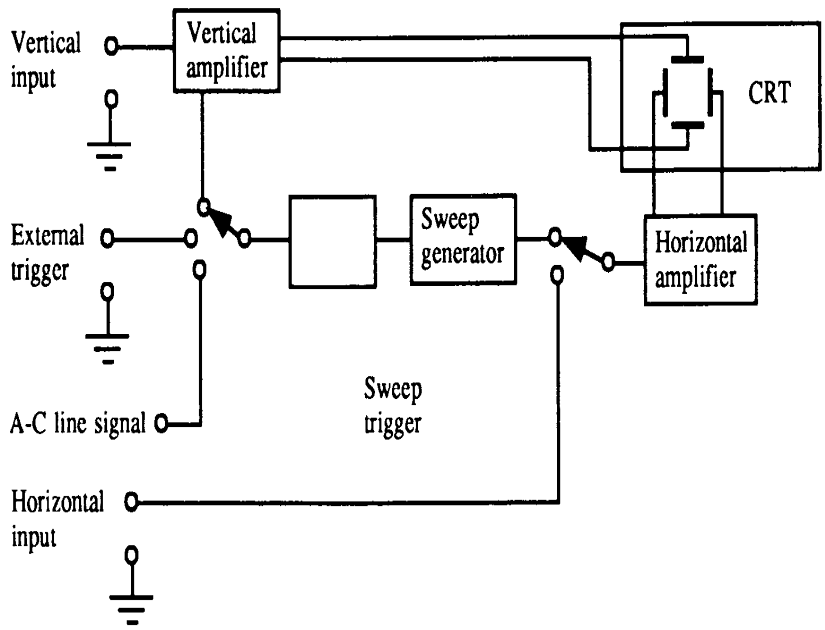


Figure 6 Block diagram of a typical oscilloscope

2.2 Ammeter usage

2.2.1 Introduction

Parts and Materials

- 6-volt battery
- 6-volt incandescent lamp

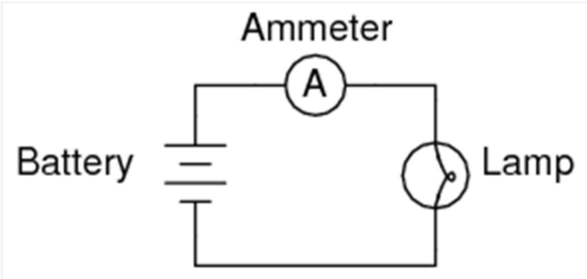
Basic circuit construction components such as breadboard, terminal strip, and jumper wires are also assumed to be available from now on, leaving only components and materials unique to the project listed under "Parts and Materials."

Cross-References

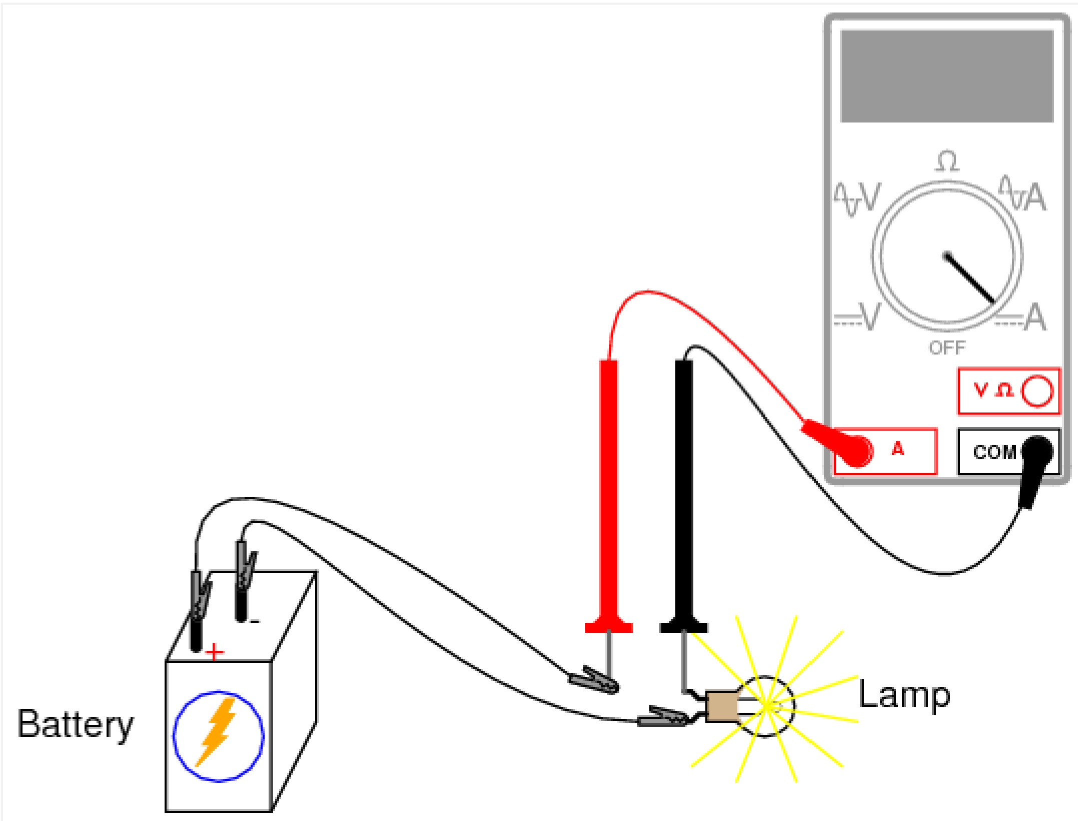
Lessons In Electric Circuits, Volume 1, chapter 1: "Basic Concepts of Electricity"

Lessons In Electric Circuits, Volume 1, chapter 8: "DC Metering Circuits"

Schematic Diagram



Illustration



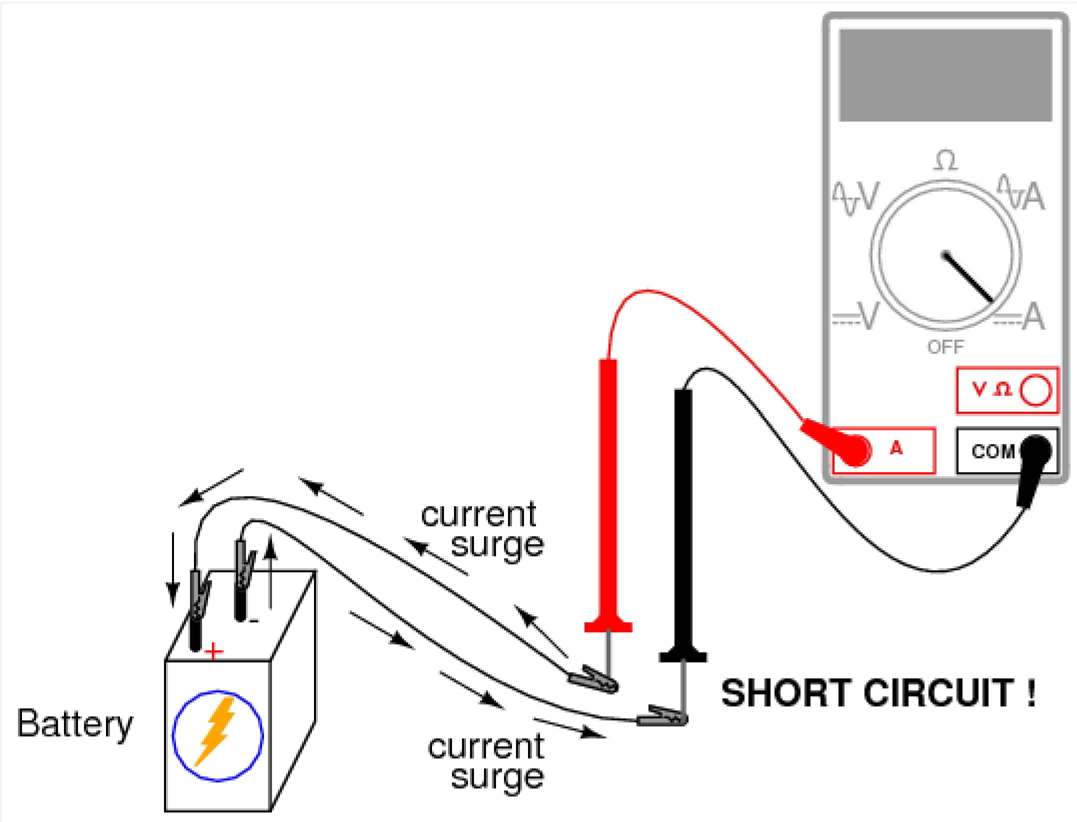
2.2.2 Instructions

Current is the measure of the rate of electron "flow" in a circuit. It is measured in the unit of the Ampere, simply called "Amp," (A).

The most common way to measure current in a circuit is to break the circuit open and insert an "ammeter" in *series* (in-line) with the circuit so that all electrons flowing through the circuit also have to go through the meter. Because measuring current in this manner requires the meter be made part of the circuit, it is a more difficult type of measurement to make than either voltage or resistance.

Some digital meters, like the unit shown in the illustration, have a separate jack to insert the red test lead plug when measuring current. Other meters, like most inexpensive analog meters, use the same jacks for measuring voltage, resistance, and current. Consult your owner's manual on the particular model of meter you own for details on measuring current.

When an ammeter is placed in series with a circuit, it ideally drops no voltage as current goes through it. In other words, it acts very much like a piece of wire, with very little resistance from one test probe to the other. Consequently, an ammeter will act as a short circuit if placed in parallel (across the terminals of) a substantial source of voltage. If this is done, a surge in current will result, potentially damaging the meter:



Ammeters are generally protected from excessive current by means of a small *fuse* located inside the meter housing. If the ammeter is accidently connected across a substantial voltage source, the resultant surge in current will "blow" the fuse and render the meter incapable of measuring current until the fuse is replaced. **Be very careful to avoid this scenario!**

You may test the condition of a multi meter's fuse by switching it to the resistance mode and measuring continuity through the test leads (and through the fuse). On a meter where the same test lead jacks are used for both resistance and current measurement, simply leave the test lead plugs where they are and touch the two probes together. On a meter where different jacks are used, this is how you insert the test lead plugs to check the fuse:

2.2.3 Ammeter

An **ammeter** is a measuring instrument used to measure the electric current in a circuit. Electric currents are measured in amperes (A), hence the name. Instruments used to measure smaller currents, in the milliampere or microampere range, are designated as *milliammeters* or *microammeters*. Early ammeters were laboratory instruments which relied on the Earth's magnetic field for operation. By the late 19th century, improved instruments were designed which could be mounted in any position and allowed accurate measurements in electric power systems.

2.2.3.1 History

The relation between electric current, magnetic fields and physical forces was first noted by Hans Christian Ørsted who, in 1820, observed a compass needle was deflected from pointing north when a current flowed in an adjacent wire. The tangent galvanometer was used to measure currents using this effect, where the restoring force returning the pointer to the zero position was provided by the Earth's magnetic field. This made these instruments usable only when aligned with the Earth's field. Sensitivity of the instrument was increased by using additional turns of wire to multiply the effect – the instruments were called "multipliers".

2.2.3.2 Types

The D'Arsonval galvanometer is a **moving coil** ammeter. It uses magnetic deflection, where current passing through a coil causes the coil to move in a magnetic field. The voltage drop across the coil is kept to a minimum to minimize resistance across the ammeter in any circuit into which it is inserted. The modern form of this instrument was developed by Edward Weston, and uses two spiral springs to provide the restoring force. By maintaining a uniform air gap between the iron core of the instrument and the poles of its permanent magnet, the instrument has good linearity and accuracy. Basic meter movements can have full-scale deflection for currents from about 25 microamperes to 10 milli amperes and have linear scales.

Moving iron: ammeters use a piece of iron which moves when acted upon by the electromagnetic force of a fixed coil of wire. This type of meter responds to both-direct and alternating currents (as opposed to the moving coil ammeter, which works on direct current only). The iron element consists of a moving vane attached to a pointer, and a fixed vane, surrounded by a coil. As alternating or direct current flows through the coil and induces a magnetic field in both vanes, the vanes repel each other and the moving vane deflects against the restoring force provided by fine helical springs. The non-linear scale of these meters makes them unpopular.

3 AC Circuits (Chapter 6)

Learning Objectives

After going through this chapter you will be able to

- describe the relationship between sinusoidal voltages and currents in resistors, capacitors and inductors
- explain the meaning of terms such as reactance and impedance, and calculate values for these quantities for individual components and simple circuits
- analyse circuits containing resistors, capacitors and inductors to determine the associated voltages and currents
- explain the use of complex numbers in the description and analysis of circuit behaviour.

3.1 Introduction

"AC" stands for **A**lternating **C**urrent, which can refer to either voltage or current that alternates in polarity or direction, respectively. A convenient source of AC voltage is household wall-socket power, which presents significant shock hazard. In order to minimize this hazard while taking advantage of the convenience of this source of AC, a small *power supply* will be the first project, consisting of a *transformer* that steps the hazardous voltage (110 or more volts AC, RMS) down to 12 volts or less. The title of "power supply" is somewhat misleading. This device does not really act as a source or *supply* of power, but rather as a power *converter*, to reduce the hazardous voltage of wall-socket power to a much safer level.

AC through Resistors

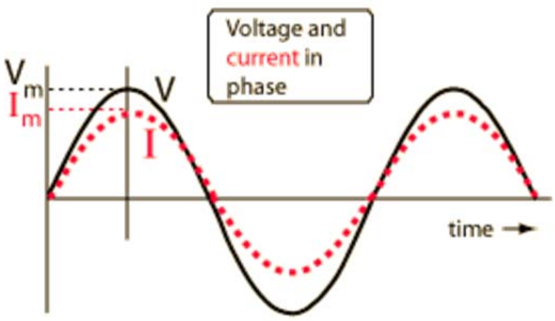
Resistor AC Response

Impedance

$$I = \frac{V}{R}$$

$$Z = R$$

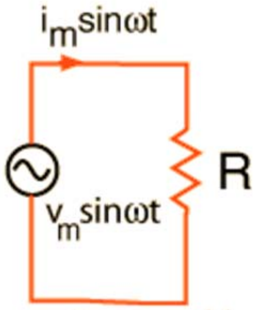
Calculate



Examine

Capacitor

Inductor



$$I = \frac{I_m}{\sqrt{2}}, V = \frac{V_m}{\sqrt{2}}$$

Contribution to complex impedance	Phasor diagram
R	

For ordinary currents and frequencies the behaviour of a resistor is that of a dissipative element which converts electrical energy into heat. It is independent of the direction of current flow and independent of the frequency. So we say that the AC impedance of a resistor is the same as its DC resistance. That assumes, however, that you are using the rms or effective values for the current and voltage in the AC case.

3.2 AC capacitor circuits

Capacitors do not behave the same as resistors. Whereas resistors allow a flow of electrons through them directly proportional to the voltage drop, capacitors oppose *changes* in voltage by drawing or supplying current as they charge or discharge to the new voltage level. The flow of electrons "through" a capacitor is directly proportional to the *rate of change* of voltage across the capacitor. This opposition to voltage change is another form of *reactance*, but one that is precisely opposite to the kind exhibited by inductors.

Expressed mathematically, the relationship between the current "through" the capacitor and rate of voltage change across the capacitor is as such:

$$i = C\frac{de}{dt}$$

The expression *de/dt* is one from calculus, meaning the rate of change of instantaneous voltage (e) over time, in volts per second. The capacitance (C) is in Farads, and the instantaneous current (i), of course, is in amps. Sometimes you will find the rate of instantaneous voltage change over time expressed as dv/dt instead of de/dt: using the lower-case letter "v" instead or "e" to represent voltage, but it means the exact same thing. To show what happens with alternating current, let's analyze a simple capacitor circuit: (Figure 7).

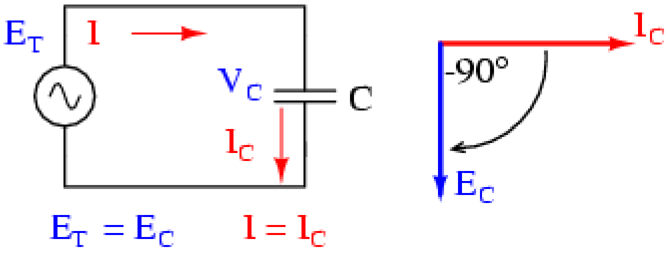


Figure 7 Pure capacitive circuit: capacitor voltage lags capacitor current by 90°

If we were to plot the current and voltage for this very simple circuit, it would look something like this: (Figure 8)

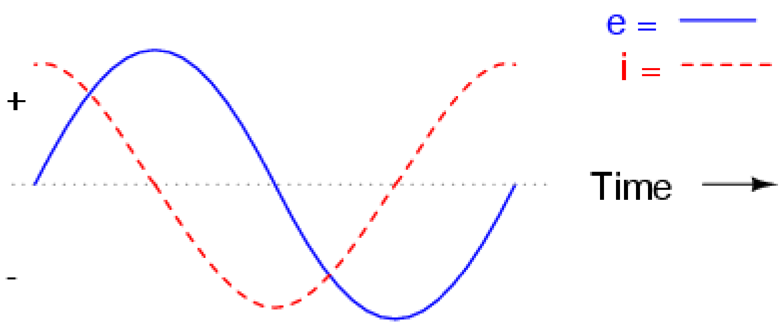


Figure 8 Pure capacitive circuit waveforms.

Remember, the current through a capacitor is a reaction against the *change* in voltage across it. Therefore, the instantaneous current is zero whenever the instantaneous voltage is at a peak (zero change, or level slope, on the voltage sine wave), and the instantaneous current is at a peak wherever the instantaneous voltage is at maximum change (the points of steepest slope on the voltage wave, where it crosses the zero line). This results in a voltage wave that is -90°out of phase with the current wave. Looking at the graph, the current wave seems to have a "head start" on the voltage wave; the current "leads" the voltage, and the voltage "lags" behind the current. (Figure 9)

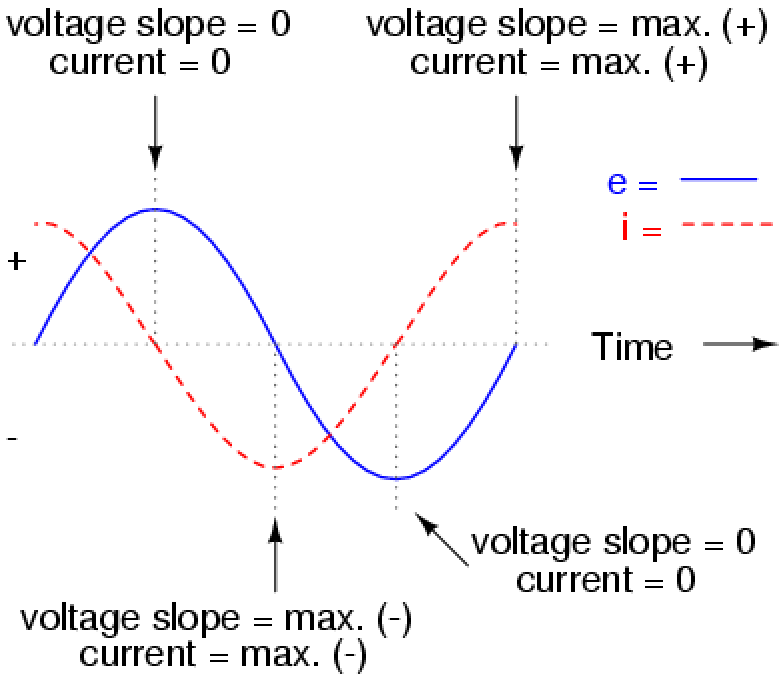


Figure 9 Voltage lags current by 90º in a pure capacitive circuit.

As you might have guessed, the same unusual power wave that we saw with the simple inductor circuit is present in the simple capacitor circuit, too: (Figure 10)

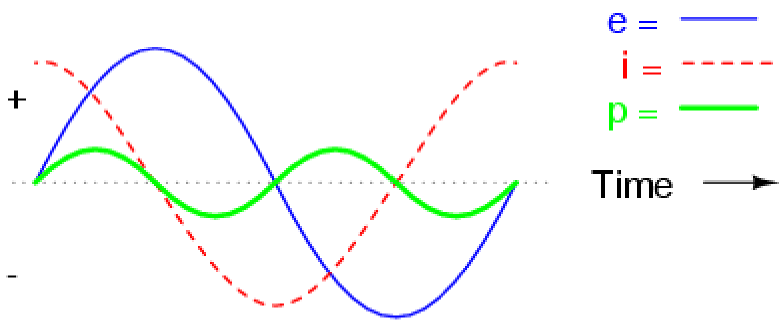


Figure 10 In a pure capacitive circuit, the instantaneous power may be positive or negative.

As with the simple inductor circuit, the 90 degree phase shift between voltage and current results in a power wave that alternates equally between positive and negative. This means that a capacitor does not dissipate power as it reacts against changes in voltage; it merely absorbs and releases power, alternately.

A capacitor's opposition to change in voltage translates to an opposition to alternating voltage in general, which is by definition always changing in instantaneous magnitude and direction. For any given magnitude of AC voltage at a given frequency, a capacitor of given size will "conduct" a certain magnitude of AC current. Just as the current through a resistor is a function of the voltage across the resistor and the resistance offered by the resistor, the AC current through a capacitor is a function of the AC voltage across it, and the *reactance* offered by the capacitor. As with inductors, the reactance of a capacitor is expressed in ohms and symbolized by the letter X (or X_C to be more specific).

Since capacitors "conduct" current in proportion to the rate of voltage change, they will pass more current for faster-changing voltages (as they charge and discharge to the same voltage peaks in less time), and less current for slower-changing voltages. What this means is that reactance in ohms for any capacitor is *inversely* proportional to the frequency of the alternating current. (Table below)

$$X_C = \frac{1}{2\pi f C}$$

3.3 Reactance of a 100 uF capacitor:

Frequency (Hertz)	Reactance (Ohms)
60	26.5258
120	13.2629
2500	0.6366

Please note that the relationship of capacitive reactance to frequency is exactly opposite from that of inductive reactance. Capacitive reactance (in ohms) decreases with increasing AC frequency. Conversely, inductive reactance (in ohms) increases with increasing AC frequency. Inductors oppose faster changing currents by producing greater voltage drops; capacitors oppose faster changing voltage drops by allowing greater currents.

As with inductors, the reactance equation's $2\pi f$ term may be replaced by the lower-case Greek letter Omega (ω), which is referred to as the *angular velocity* of the AC circuit. Thus, the equation $X_C = 1/(2\pi fC)$ could also be written as $X_C = 1/(\omega C)$, with ω cast in units of *radians per second*.

Alternating current in a simple capacitive circuit is equal to the voltage (in volts) divided by the capacitive reactance (in ohms), just as either alternating or direct current in a simple resistive circuit is equal to the voltage (in volts) divided by the resistance (in ohms). The following circuit illustrates this mathematical relationship by example: (Figure 11)

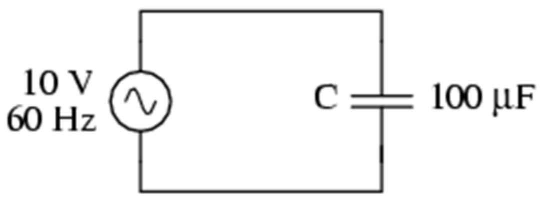


Figure 11 Capacitive reactance.

$$X_C = 26.5258\ \Omega$$

$$I = \frac{E}{X}$$

$$I = \frac{10\ V}{26.5258\ \Omega}$$

$$I = 0.3770\ A$$

However, we need to keep in mind that voltage and current are not in phase here. As was shown earlier, the current has a phase shift of +90º with respect to the voltage. If we represent these phase angles of voltage and current mathematically, we can calculate the phase angle of the capacitor's reactive opposition to current.

$$\text{Opposition} = \frac{\text{Voltage}}{\text{Current}}$$

$$\text{Opposition} = \frac{10\ V \angle 0^\circ}{0.3770\ A \angle 90^\circ}$$

$$\text{Opposition} = 26.5258\ \Omega \angle -90^\circ$$

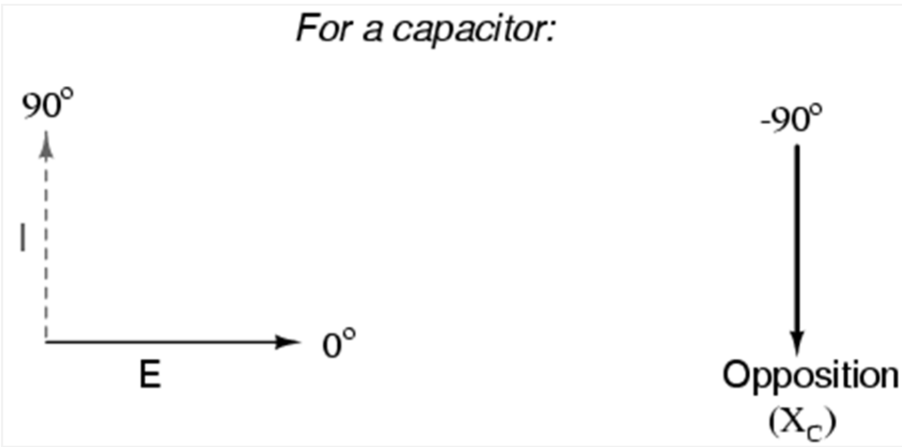


Figure 12 Voltage lags current by 90º in a capacitor.

Mathematically, we say that the phase angle of a capacitor's opposition to current is -90º, meaning that a capacitor's opposition to current is a negative imaginary quantity. (Figure 12) This phase angle of reactive opposition to current becomes critically important in circuit analysis, especially for complex AC circuits where reactance and resistance interact.

It will prove beneficial to represent *any* component's opposition to current in terms of complex numbers, and not just scalar quantities of resistance and reactance.

REVIEW

- *Capacitive reactance* is the opposition that a capacitor offers to alternating current due to its phase-shifted storage and release of energy in its electric field. Reactance is symbolized by the capital letter "X" and is measured in ohms just like resistance (R).
- Capacitive reactance can be calculated using this formula: $X_C = 1/(2\pi fC)$
- Capacitive reactance *decreases* with increasing frequency. In other words, the higher the frequency, the less it opposes (the more it "conducts") the AC flow of electrons.

3.4 Resistors in AC Circuits

In the previous tutorials we have looked at resistors, their connections and used Ohm's Law to calculate the voltage, current and power associated with them. In all cases both the voltage and current has been assumed to be of a constant polarity, flow and direction, in other words Direct Current or DC.

But there is another type of supply known as Alternating Current or **AC** whose voltage switches polarity from positive to negative and back again over time and also whose current with respect to the voltage oscillates back and forth. The oscillating shape of an AC supply follows that of the mathematical form of a "sine wave" which is commonly called a **Sinusoidal Waveform**. Therefore, a sinusoidal voltage can be defined as $V(t) = V_{max} \sin \omega t$.

When using pure resistors in AC circuits that have negligible values of inductance or capacitance, the same principals of Ohm's Law, circuit rules for voltage, current and power (and even Kirchhoff's Laws) apply as they do for DC resistive circuits the only difference this time is in the use of the instantaneous "peak-to-peak" or "rms" quantities.

When working with AC alternating voltages and currents it is usual to use only "rms" values to avoid confusion. Also the schematic symbol used for defining an AC voltage source is that of a "wavy" line as opposed to a battery symbol for DC and this is shown in Figure 13.

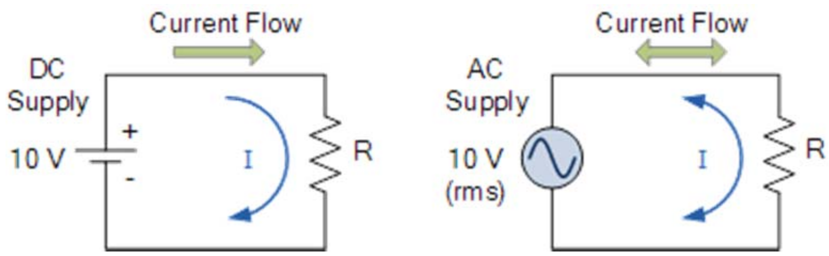


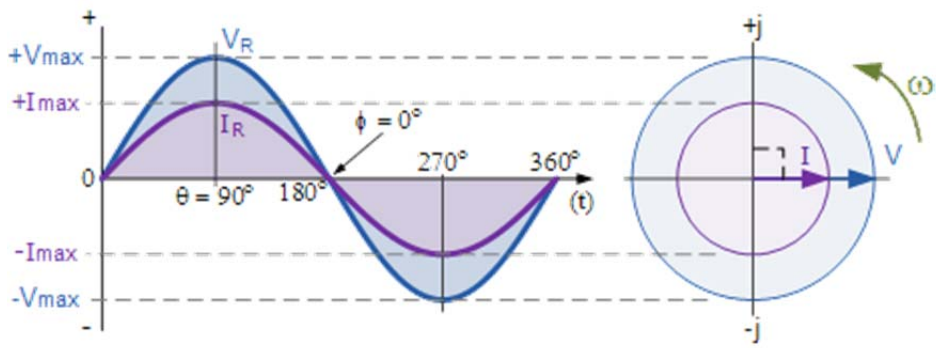
Figure 13 Symbol Representation of DC and AC Supplies

Resistors are "passive" devices, which mean they do not produce or consume any electrical energy, but convert electrical energy into heat. In DC circuits the linear ratio of voltage to current in a resistor is called its resistance. However, in AC circuits this ratio of voltage to current depends upon the frequency and phase difference or phase angle (ϕ) of the supply. So when using resistors in AC circuits the term **Impedance**, symbol **Z** is the generally used and we can say that DC resistance = AC impedance, R = Z.

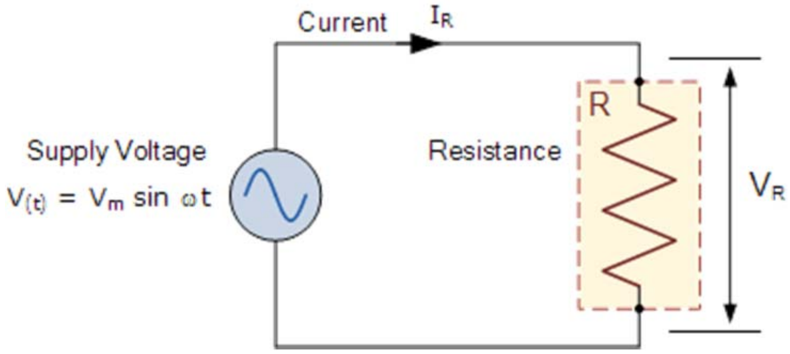
It is important to note, that when used in AC circuits, a resistor will always have the same resistive value no matter what the supply frequency from DC to very high frequencies, unlike capacitor and inductors.

For resistors in AC circuits the direction of the current flowing through them has no effect on the behaviour of the resistor so will rise and fall as the voltage rises and falls. The current and voltage reach maximum, fall through zero and reach minimum at exactly the same time. i.e., they rise and fall simultaneously and are said to be "in-phase" as shown below.

3.4.1 V-I Phase Relationship and Vector Diagram



We can see that at any point along the horizontal axis that the instantaneous voltage and current are in-phase because the current and the voltage reach their maximum values at the same time that is their phase angle θ is 0°.Then these instantaneous values of voltage and current can be compared to give the ohmic value of the resistance simply by using ohms law. Consider below the circuit consisting of an AC source and a resistor.



The instantaneous voltage across the resistor, V_R is equal to the supply voltage, V_t and is given as:

$$V_R = V_{max} \sin \omega t$$

The instantaneous current flowing in the resistor will therefore be:

$$I_R = \frac{V_R}{R} = \frac{V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

As the voltage across a resistor is given as $V_R = I.R$, the instantaneous voltage across the resistor above can also be given as:

$$V_R = I_{max} R \sin \omega t$$

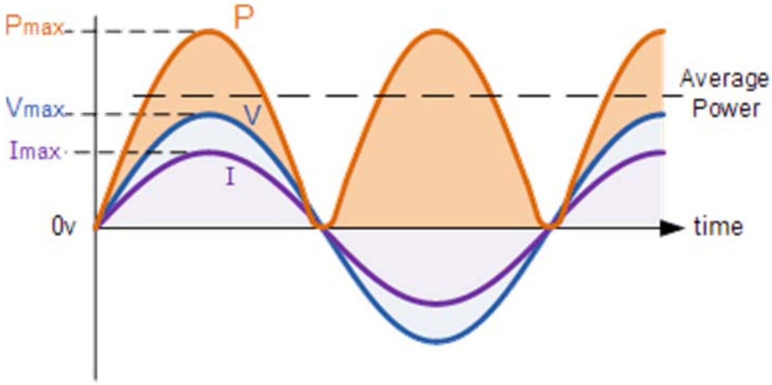
In purely resistive series AC circuits, all the voltage drops across the resistors can be added together to find the total circuit voltage as all the voltages are in-phase with each other. Likewise, in a purely resistive parallel AC circuit, all the individual branch currents can be added together to find the total circuit current because all the branch currents are in-phase with each other.

Since for resistors in AC circuits the phase angle φ between the voltage and the current is zero, then the power factor of the circuit is given as $\cos 0^o = 1.0$. The power in the circuit at any instant in time can be found by multiplying the voltage and current at that instant.

Then the power (P), consumed by the circuit is given as $P = Vrms\ I \cos \Phi$ in watt's. But since $\cos \Phi = 1$ in a purely resistive circuit, the power consumed is simply given as, $P = Vrms\ I$ the same as for Ohm's Law.

This then gives us the "Power" waveform and which is shown below as a series of positive pulses because when the voltage and current are both in their positive half of the cycle the resultant power is positive. When the voltage and current are both negative, the product of the two negative values gives a positive power pulse.

3.4.2 Power Waveform in a Pure Resistance



Then the power dissipated in a purely resistive load fed from an AC rms supply is the same as that for a resistor connected to a DC supply and is given as:

$$P = V_{R(rms)} \times I_{rms} = I^2_{rms} R = \frac{V^2_{rms}}{R}$$

Where:

- P is the average power in Watts
- V_{rms} is the rms supply voltage in Volts
- I_{rms} is the rms supply current in Amps
- R is the resistance of the resistor in Ohm's (Ω) – should really be Z to indicate impedance

The heating effect produced by an AC current with a maximum value of Imax is not the same as that of a DC current of the same value. To compare the AC heating effect to an equivalent DC the rms values must be used. Any resistive heating element such as Electric Fires, Toasters, Kettles, Irons, and Water Heaters etc can be classed as a resistive AC circuit and we use resistors in AC circuits to heat our homes and water.

1. Example: Resistors in AC Circuits

A 1000W heating element is connected to a 250v AC supply voltage. Calculate the impedance (AC resistance) of the element when it is hot and the amount of current taken from the supply.

$$\text{Current}, I = \frac{P}{V} = \frac{1000W}{250V} = 4 \text{ amps}$$

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\Omega$$

2. Example: Resistors in AC Circuits

Calculate the power being consumed by a 100Ω resistive element connected across a 240v supply.

As there is only one component connected to the supply, the resistor, then $V_R = V_S$

$$\text{Current, } I = \frac{V_R}{R} = \frac{240}{100} = 2.4 \text{ amps}$$

$$\text{Power consumed, } P = I^2 R = 2.4^2 \times 100 = 576W$$

Then to summarise, in a pure ohmic AC Resistance, the current and voltage are both said to be "in-phase" as there is no phase difference between them. The current flowing through the resistor is directly proportional to the voltage across it with this linear relationship in an AC circuit being called Impedance. As with DC circuits, Ohm's Law can be used when working with resistors in AC circuits to calculate the resistors voltages, currents and power.

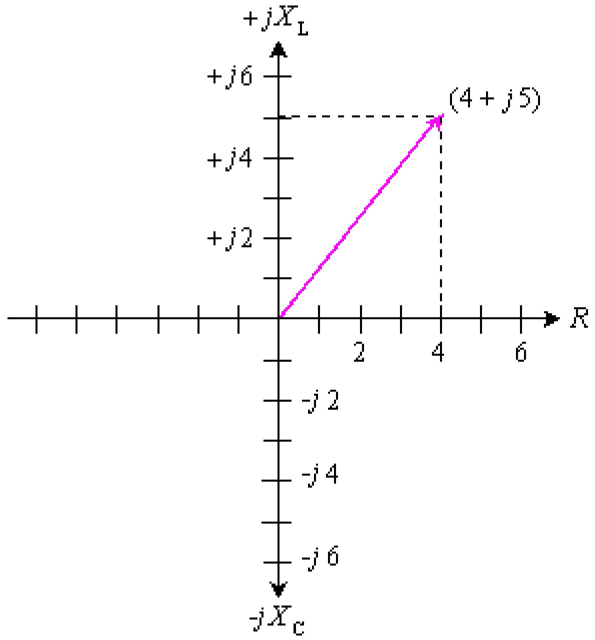
3.5 Impedance

Impedance, denoted *Z*, is an expression of the opposition that an electronic component, circuit, or system offers to alternate and/or direct electric current. Impedance is a vector (two-dimensional) quantity consisting of two independent scalar (one-dimensional) phenomena: resistance and reactance.

Resistance, denoted *R*, is a measure of the extent to which a substance opposes the movement of electrons among its atoms. The more easily the atoms give up and/or accept electrons, the lower the resistance, which is expressed in positive real number ohms. Resistance is observed with alternating current (AC) and also with direct current (DC). Examples of materials with low resistance, known as electrical conductors, include copper, silver, and gold. High-resistance substances are called insulators or dielectrics, and include materials such as polyethylene, mica, and glass. A material with an intermediate level of resistance is classified as a semiconductor. Examples are silicon, germanium, and gallium arsenide.

Reactance, denoted *X*, is an expression of the extent to which an electronic component, circuit, or system stores and releases energy as the current and voltage fluctuate with each AC cycle. Reactance is expressed in imaginary number ohms. It is observed for AC, but not for DC. When AC passes through a component that contains reactance, energy might be stored and released in the form of a magnetic field, in which case the reactance is inductive (denoted $+jX_L$); or energy might be stored and released in the form of an electric field, in which case the reactance is capacitive (denoted $-jX_C$). Reactance is conventionally multiplied by the positive square root of -1, which is the unit imaginary number called the *j operator*, to express *Z* as a complex number of the form $R + jX_L$ (when the net reactance is inductive) or $R - jX_C$ (when the net reactance is capacitive).

The illustration shows a coordinate plane modified to denote complex-number impedances. Resistance appears on the horizontal axis, moving toward the right.(The left-hand half of this coordinate plane is not normally used because negative resistances are not encountered in common practice.)Inductive reactance appears on the positive imaginary axis, moving upward. Capacitive reactance is depicted on the negative imaginary axis, moving downward. As an example, a complex impedance consisting of 4 ohms of resistance and +*j*5 ohms of inductive reactance is denoted as a vector from the origin to the point on the plane corresponding to 4 + *j*5.



In series circuits, resistances and reactances add together independently. Suppose a resistance of 100.00 ohms is connected in a series circuit with an inductance of 10.000 ?H. At 4.0000 MHz, the complex impedance is:

$Z_{RL} = R + jX_L = 100.00 + j251.33$

If a capacitor of 0.0010000 ?F is put in place of the inductor, the resulting complex impedance at 4.0000 MHz is:

$Z_{RC} = R - jX_C = 100.00 - j39.789$

If all three components are connected in series, then the reactances add, yielding a complex impedance of:

$Z_{RLC} = 100 + j251.33 - j39.789 = 100 + j211.5$

This is the equivalent of a 100-ohm resistor in series with an inductor having +*j*211.5 ohms of reactance. At 4.0000 MHz, this reactance is presented by an inductance of 8.415 ?H, as determined by plugging the numbers into the formula for inductive reactance and working backwards.(See the definition of for this formula, and for the corresponding formula for capacitive reactance.)

Parallel RLC circuits are more complicated to analyze than are series circuits. To calculate the effects of capacitive and inductive reactance in parallel, the quantities are converted to *inductive susceptance* and *capacitive susceptance*. Susceptance is the reciprocal of reactance. Susceptance combines with *conductance*, which is the reciprocal of resistance, to form *complex admittance*, which is the reciprocal of complex impedance. Entire volumes have been devoted to the theoretical and practical aspects of resistance, conductance, reactance, susceptance, impedance, and admittance. An intermediate electronics text or reference book is recommended for further study.

4 Capacitors (Chapter 8)

Learning Objectives

After going through this chapter you will be able to

- define capacitance and state its symbol and unit of measurement;
- predict the capacitance of a parallel plate capacitor;
- analyze how a capacitor stores energy;
- define inductance and state its symbol and unit of measurement;
- predict the inductance of a coil of wire.

4.1 Introduction

A **capacitor** (originally known as a **condenser**) is a passive two-terminal electrical component used to store energy electro statically in an electric field. The forms of practical capacitors vary widely, but all contain at least two electrical conductors (plates) separated by a dielectric (i.e., insulator). The conductors can be thin films of metal, aluminium foil or disks, etc. The 'non conducting' dielectric acts to increase the capacitor's charge capacity. A dielectric can be glass, ceramic, plastic film, air, paper, mica, etc. Capacitors are widely used as parts of electrical circuits in many common electrical devices. Unlike a resistor, a capacitor does not dissipate energy. Instead, a capacitor stores energy in the form of an electrostatic field between its plates.

When there is a potential difference across the conductors (e.g., when a capacitor is attached across a battery), an electric field develops across the dielectric, causing positive charge (+Q) to collect on one plate and negative charge (-Q) to collect on the other plate. If a battery has been attached to a capacitor for a sufficient amount of time, no current can flow through the capacitor. However, if an accelerating or alternating voltage is applied across the leads of the capacitor, a displacement current can flow.

An ideal capacitor is characterized by a single constant value for its capacitance. Capacitance is expressed as the ratio of the electric charge (Q) on each conductor to the potential difference (V) between them. The SI unit of capacitance is the farad (F), which is equal to one coulomb per volt (1 C/V). Typical capacitance values range from about 1 pF (10^{-12} F) to about 1 mF (10^{-3} F).

The capacitance is greater when there is a narrower separation between conductors and when the conductors have a larger surface area. In practice, the dielectric between the plates passes a small amount of leakage current and also has an electric field strength limit, known as the breakdown voltage. The conductors and leads introduce an undesired inductance and resistance.

Capacitors are widely used in electronic circuits for blocking direct current while allow in alternating current to pass. In analog filter networks, they smooth the output of power supplies. In resonant circuits they tune radios to particular frequencies. In electric power transmission systems they stabilize voltage and power flow.

A capacitor consists of two conductors separated by a non-conductive region. The non-conductive region is called the dielectric. In simpler terms, the dielectric is just an electrical insulator. Examples of dielectric media are glass, air, paper, vacuum, and even a semi-conductor-depletion region chemically identical to the conductors. A capacitor is assumed to be self-contained and isolated, with no net electric charge and no influence from any external electric field. The conductors thus hold equal and opposite charges on their facing surfaces, and the dielectric develops an electric field. In SI units, a capacitance of one farad means that one coulomb of charge on each conductor causes a voltage of one volt across the device.

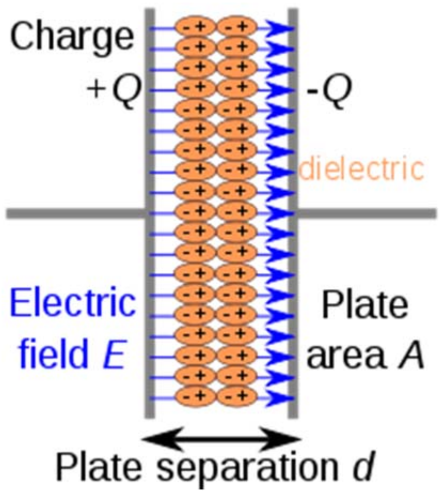
An ideal capacitor is wholly characterized by a constant capacitance C, defined as the ratio of charge $\pm Q$ on each conductor to the voltage V between them:

$$C = \frac{Q}{V}$$

Because the conductors (or plates) are close together, the opposite charges on the conductors attract one another due to their electric fields, allowing the capacitor to store more charge for a given voltage than if the conductors were separated, giving the capacitor a large capacitance.

Sometimes charge build-up affects the capacitor mechanically, causing its capacitance to vary. In this case, capacitance is defined in terms of incremental changes:

$$C = \frac{\mathrm{d}Q}{\mathrm{d}V}$$



4.2 Energy of electric field

Work must be done by an external influence to "move" charge between the conductors in a capacitor. When the external influence is removed, the charge separation persists in the electric field and energy is stored to be released when the charge is allowed to return to its equilibrium position. The work done in establishing the electric field, and hence the amount of energy stored, is

$$W = \int_0^Q V \mathrm{d}q = \int_0^Q \frac{q}{C} \mathrm{d}q = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}VQ$$

Here Q is the charge stored in the capacitor, V is the voltage across the capacitor, and C is the capacitance.

In the case of a fluctuating voltage $V(t)$, the stored energy also fluctuates and hence power must flow into or out of the capacitor. This power can be found by taking the time derivative of the stored energy:

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}CV^2\right) = CV(t)\frac{\mathrm{d}V}{\mathrm{d}t}$$

4.3 Networks

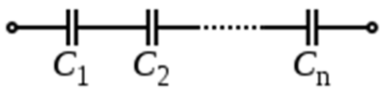
Series and parallel circuits

4.3.1 For capacitors in parallel

Capacitors in a parallel configuration each have the same applied voltage. Their capacitances add up. Charge is apportioned among them by size. Using the schematic diagram to visualize parallel plates, it is apparent that each capacitor contributes to the total surface area.

$$C_{\mathrm{eq}} = C_1 + C_2 + \cdots + C_n$$

4.3.2 For capacitors in series



Connected in series, the schematic diagram reveals that the separation distance, not the plate area, adds up. The capacitors each store instantaneous charge build-up equal to that of every other capacitor in the series. The total voltage difference from end to end is apportioned to each capacitor according to the inverse of its capacitance. The entire series acts as a capacitor *smaller* than any of its components.

$$\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

Capacitors are combined in series to achieve a higher working voltage, for example for smoothing a high voltage power supply. The voltage ratings, which are based on plate separation, add up, if capacitance and leakage currents for each capacitor are identical. In such an application, on occasion series strings are connected in parallel, forming a matrix. The goal is to maximize the energy storage of the network without overloading any capacitor. For high-energy storage with capacitors in series, some safety considerations must be applied to ensure one capacitor failing and leaking current will not apply too much voltage to the other series capacitors.

4.3.3 Voltage distribution in parallel-to-series networks

To model the distribution of voltages from a single charged capacitor (A) connected in parallel to a chain of capacitors in series (B_{n}):

$$(volts)A_{\mathrm{eq}} = A\left(1 - \frac{1}{n+1}\right)$$
$$(volts)B_{1..\mathrm{n}} = \frac{A}{n}\left(1 - \frac{1}{n+1}\right)$$
$$A - B = 0$$

Note: This is only correct if all capacitance values are equal.

The power transferred in this arrangement is:

$$P = \frac{1}{R} \cdot \frac{1}{n+1} A_{\mathrm{volts}} \left(A_{\mathrm{farads}} + B_{\mathrm{farads}}\right)$$

Series connection is also sometimes used to adapt polarized electrolytic capacitors for bipolar AC use. Two identical polarized electrolytic capacitors are connected back to back to form a bipolar capacitor with half the nominal capacitance of either. However, the anode film can only withstand a small reverse voltage. This arrangement can lead to premature failure as the anode film is broken down during the reverse-conduction phase and partially rebuilt during the forward phase. A factory-made non-polarized electrolytic capacitor has both plates anodized so that it can withstand rated voltage in both directions; such capacitors also have about half the capacitance per unit volume of polarized capacitors.

4.4 Coulomb's Law

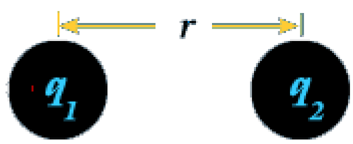
The magnitude of the force of attraction or repulsion between two electric charges at rest was studied by Charles Coulomb. He formulated a law, known as "COULOMB'S LAW".

4.4.1 Statement

- According to Coulomb's law:
  The electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of charges.
- The electrostatic force of attraction or repulsion between two point charges is inversely proportional to the square of distance between them.

4.4.2 Mathematical Representation of Coulomb's Law[1]

Consider two point charges **q1** and **q2** placed at a distance of **r** from each other. Let the electrostatic force between them is **F**.



According to the first part of the law: $F \propto q_1 q_2$

According to the second part of the law: $F \propto \frac{1}{r^2}$

Combining above statements: $F \propto \frac{q_1 q_2}{r^2}$

Or: $$F = K\frac{q_1 q_2}{r^2}$$

(Where k is the constant of proportionality)

Value of K[2]

Value of K is equal to 1/4pe0

where pe0 is permittivity of free space .Its volume is 8.85 x 10-12 c2/Nm2.

Thus in S.I. system numerical value of K is 8.98755 x 109 Nm2c-2.

[1] www.citycollegiate.com
[2] www.citycollegiate.com

4.4.3 Other Forms of Coulomb's Law

Putting the value of **K = 1/4**pe0 in equation (i)

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

4.4.4 Force to the Presence of Dielectric Medium[3]

If the space between the charges is filled with a non conducting medium or an insulator called "dielectric", it is found that the dielectric reduces the electrostatic force as compared to free space by a factor (e_r) called DIELECTRIC CONSTANT. It is denoted by e_r. This factor is also known as RELATIVE PERMITTIVITY. It has different values for different dielectric materials.

In the presence of a dielectric between two charges the Coulomb's law is expressed as:

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}$$

4.4.5 Vector Form of Coulomb's Law

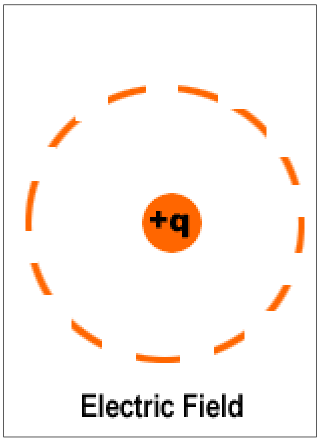
The magnitude as well as the direction of electrostatic force can be expressed by using Coulomb's law by vector equation:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Where $\vec{F}_{12}$ is the force exerted by **q1** on **q2** and $\hat{r}_{12}$ is the unit vector along the line joining the two charges from **q1** to **q2.**

4.5 Electric Field

When an electric charge is placed in space, the space around the charge is modified and if we place another test charge within this space, the test charge will experience some electrostatic force. The modified space around an electric charge is called 'ELECTRIC FIELD'.



For an exact **definition** we can describe an electric field as:

Space or region surrounding an electric charge or a charged body within which another charge experiences some electrostatic force of attraction or repulsion when placed at a point is called Electric Field.

3 www.citycollegiate.com

4.5.1 Electric Intensity

Electric Intensity is measured in the following unit: N/C or Volt/m

Definition:

Electric intensity is the strength of electric field at a point.

Electric intensity at a point is defined as the force experienced per unit positive charge at a point placed in the electric field.

It may also be also defined as the electrostatic force per unit charge which the field exerts at a point.

The force experienced by a charge +q in an electric field depends upon.

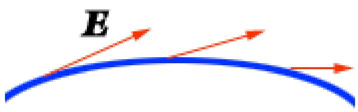
1) Magnitude of test charge (q)
2) Intensity of electric field (E)

It is a vector quantity. It has the same direction as that of force.

4.5.2 Electric Lines of Force

In order to point out the direction of an electric field we can draw a number of lines called electric lines of force.

An electric line of force is an imaginary continuous line or curve drawn in an electric field such that tangent to it at any point gives the direction of the electric force at that point. The direction of a line of force is the direction along which a small free positive charge will move along the line. It is always directed from positive charge to negative charge.

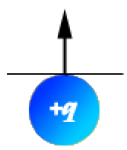


Electrical Lines of Force have the following characteristics:

Lines of force originate from a positive charge and terminate to a negative charge.

The tangent to the line of force indicates the direction of the electric field and electric force.

Electric lines of force are always normal to the surface of charged body.



Electric lines of force contract longitudinally.

Two electric lines of force cannot intersect each other.

Two electric lines of force proceeding in the same direction repel each other.

Two electric lines of force proceeding in the opposite direction attract each other.

The lines of force are imaginary but the field it represents as real.

There are no lines of force inside the conductor.

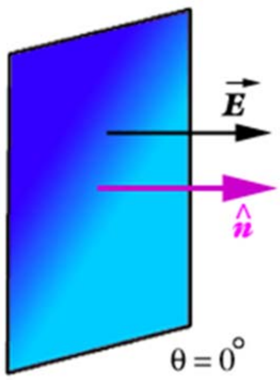
4.5.3 Electrical Flux

General Meaning of Electric Flux: In common language flux refers to the flow or stream of anything from one point to another point. In the similar way electric flux is the total number of lines of force passing through a surface.

In **physical sense**, electric flux is **defined** as "The total number of lines of force passing through the unit area of a surface held perpendicularly."

4.5.4 Maximum Flux

If the surface is placed perpendicular to the electric field then maximum electric lines of force will pass through the surface. Consequently maximum electric flux will pass through the surface.



$$\phi_e = \vec{E} \cdot \Delta\vec{A}$$

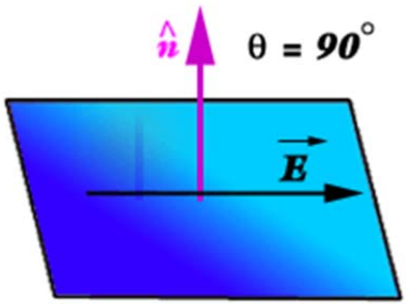
$$\phi_e = E \ \Delta A \cos 0^\circ$$

$$\phi_e = E \ \Delta A \ (1)$$

$$\boxed{\phi_e = E \ \Delta A}$$

4.5.5 Zero Flux

If the surface is placed parallel to the electric field then no electric lines of force will pass through the surface. Consequently no electric flux will pass through the the surface.



$$\phi_e = \vec{E} \cdot \vec{\Delta A}$$

$$\phi_e = E \; \Delta A \cos 90^\circ$$

$$\phi_e = E \; \Delta A \; (0)$$

$$\boxed{\phi_e = 0}$$

4.5.6 Unit of Flux

$\dfrac{\text{Newton . meter}^2}{\text{Coulomb}}$ OR Volt . meter (Flux is a scalar quantity)

5 Transformer (Chapter 11)

Objectives

After going through this chapter you will be able to

- know the basic principles of a transformer
- know the assumptions to characterize an ideal transformer
- know the relevance of the induction law.

5.1 Introduction

A **transformer** is an electrical device that transfers energy between two circuits through electromagnetic induction. A transformer may be used as a safe and efficient voltage converter to change the AC voltage at its input to a higher or lower voltage at its output. Other uses include current conversion, isolation with or without changing voltage and impedance conversion.

Pole-mounted distribution transformer with center-tapped secondary winding used to provide 'split-phase' power for residential and light commercial service, which in North America is typically rated 120/240 volt.

A transformer most commonly consists of two windings of wire that are wound around a common core to provide tight electromagnetic coupling between the windings. The core material is often a laminated iron core. The coil that receives the electrical input energy is referred to as the primary winding; the output coil is the secondary winding.

An alternating electric current flowing through the primary winding (coil) of a transformer generates a varying electromagnetic field in its surroundings which induces a vary in magnetic flux in the core of the transformer. The varying electromagnetic field in the vicinity of the secondary winding induces an electromotive force in the secondary winding, which appears as a voltage across the output terminals. If a load is connected across the secondary winding, a current flows through the secondary winding drawing power from the primary winding and its power source.

A transformer cannot operate with direct current. When connected to a DC source, a transformer typically produces a short output pulse as the input current rises.

The invention of transformers during the late 1800s enabled long distance, cheaper, and energy efficient transmission, distribution, and utilization of electrical energy. In the early days of commercial electric power, the main energy source was direct current (DC), which operates at relatively low-voltage and high-current. According to Joule's Law, energy losses are directly proportional to the square of the current. This law revealed that even a tiny decrease in current or rise in voltage can cause a substantial lowering in energy losses and costs. Thus, the historical pursuit for a high voltage low current electricity transmission system took shape. Although high voltage transmission systems offered many benefits, the future fate of high-voltage alternating current remained unclear for several reasons: high-voltage sources had a much higher risk of causing severe electrical injuries. Many essential appliances could only function at low voltage. Regarded as one of the most influential electrical innovations of all time, the introduction of transformers had successfully reduced the safety concerns associated with alternating current and had the ability to lower voltage to the value required by most essential appliances.

5.2 Transformer

5.2.1 Applications

Transformers perform voltage conversion, isolation protection, and impedance matching. In terms of voltage conversion, transformers can step up voltage and step down current from generators to high voltage transmission lines, and step down voltage/step up current to local distribution circuits or industrial customers. The step-up transformer is used to increase the secondary voltage relative to the primary voltage. The step-down transformer is used to decrease the secondary voltage relative to the primary voltage. Transformers range in size from thumbnail-sized units used in microphones to those weighing hundreds of tons interconnecting the power grid. A broad range of transformer designs are used in electronic and electric power applications, including miniature, audio, isolation, high-frequency, power conversion, etc.

5.2.2 Basic Principles

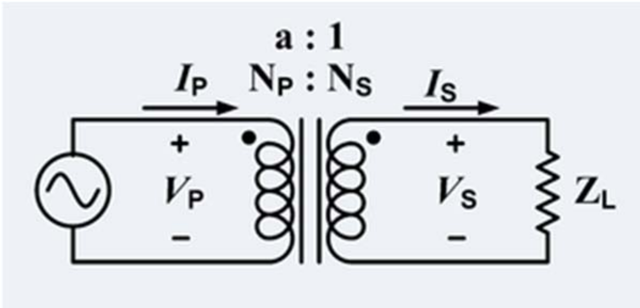
The operation of a transformer is based on two principles of the laws of electromagnetic induction: An electric current through a conductor, produces a magnetic field surrounding the conductor, and a changing magnetic field in the vicinity of a conductor induces a voltage across the ends of that conductor.

The magnetic field excited in the primary coil gives rise to self-induction as well as mutual induction between coils. This self-induction counters the excited field to such a degree that the resulting current through the primary winding is very small when the secondary winding is not connected to a load.

The physical principles of the inductive behaviour of the transformer are most readily understood and formalized when making some assumptions to construct a simple model which is called the *ideal transformer*. This model differs from *real transformers* by assuming that the transformer is perfectly constructed and by neglecting that electrical or magnetic losses occur in the materials used to construct the device.

5.2.3 Ideal Transformer

The assumptions to characterize the ideal transformer are:



Ideal transformer with a source and a load. N_P and N_S are the number of turns in the primary and secondary windings respectively.

- The windings of the transformer have no resistance. Thus, there is no copper loss in the winding, and hence no voltage drop.
- Flux is confined within the magnetic core. Therefore, it is the same flux that links the input and output windings.
- Permeability of the core is infinitely high which implies that net mmf (amp-turns) must be zero (otherwise there would be infinite flux) hence $I_P N_P - I_S N_S = 0$.
- The transformer core does not suffer magnetic hysteresis or eddy currents, which cause inductive loss.
- If the secondary winding of an ideal transformer has no load, no current flows in the primary winding.

The circuit diagram (above) shows the conventions used for an ideal, i.e. lossless and perfectly coupled transformer having primary and secondary windings with N_P and N_S turns, respectively.

The ideal transformer induces secondary voltage V_S as a proportion of the primary voltage V_P and respective winding turns as given by the equation

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = a$$

where, V_P designates source impressed voltage, V_S designates output voltage, and, "a" is the winding turns ratio, the value of these ratios being respectively higher and lower than unity for step-down and step-up transformers.

According to this formalism, when the number of turns in the primary coil is greater than the number of turns in the secondary coil, the secondary voltage is smaller than the primary voltage. On the other hand, when the number of turns in the primary coil is less than the number of turns in the secondary, the secondary voltage is greater than the primary voltage.

Any load impedance Z_L connected to the ideal transformer's secondary winding allows energy to flow without loss from primary to secondary circuits. The resulting input and output apparent power are equal as given by the equation

$$I_P V_P = I_S V_S.$$

Combining the two equations yields the following ideal transformer identity

$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = a.$$

This formula is a reasonable approximation for the typical commercial transformer, with voltage ratio and winding turns ratio both being inversely proportional to the corresponding current ratio.

The load impedance Z_L and secondary voltage V_S determine the secondary current I_S as follows

$$I_S = \frac{V_S}{Z_L}.$$

The apparent impedance Z_L' of this secondary circuit load *referred* to the primary winding circuit is governed by a squared turns ratio multiplication factor relationship derived as follows[7][8]

$$Z_L' = \frac{V_P}{I_P} = \frac{aV_S}{I_S/a} = a^2\frac{V_S}{I_S} = a^2 Z_L.$$

For an ideal transformer, the power supplied to the primary and the power dissipated by the load are equal. If $Z_L = R_L$ where R_L is a pure resistance then the power is given by:[9][10]

$$P = \frac{V_\mathrm{S}^2}{R_\mathrm{L}} = \frac{V_\mathrm{P}^2}{a^2 R_\mathrm{L}}$$

The primary current is given by the following equation:[9][10]

$$I_\mathrm{P} = \frac{V_\mathrm{P}}{a^2 Z_L}$$

5.2.4 Induction law

A varying electrical current passing through the primary coil creates a varying magnetic field around the coil which induces a voltage in the secondary winding. The primary and secondary windings are wrapped around a core of very high magnetic permeability, usually iron,[c]so that most of the magnetic flux passes through both the primary and secondary coils. The current through a load connected to the secondary winding and the voltage across it flow in the directions indicated in Figure 14.

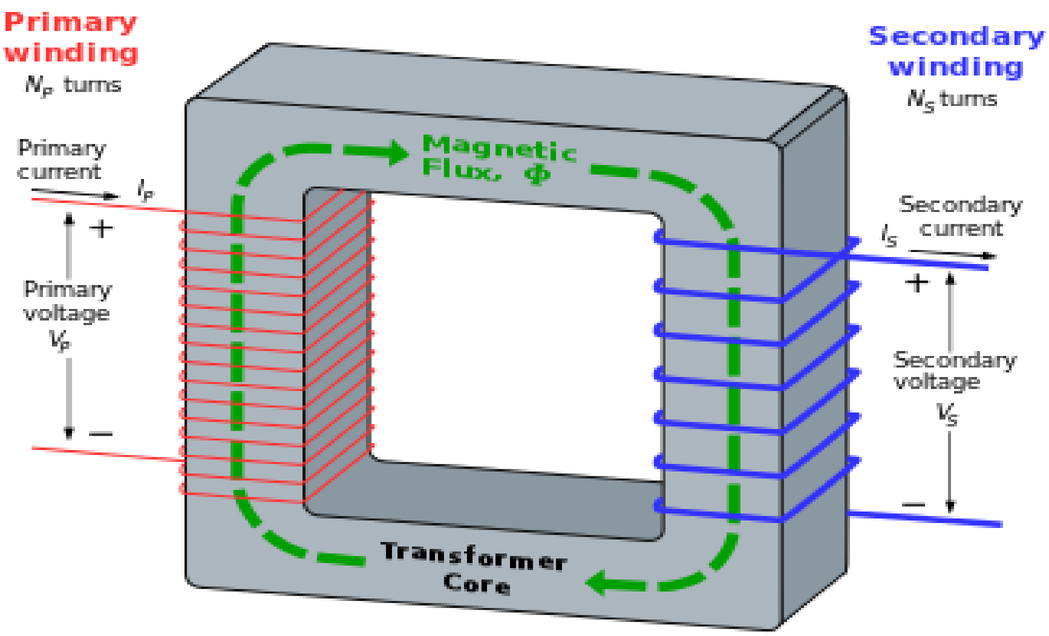


Figure 14 Transformer

5.2.5 Ideal transformer and induction law

The voltage induced across the secondary coil may be calculated from Faraday's law of induction, which states that:

$$V_S = N_S \frac{d\Phi}{dt}.$$

where V_s is the instantaneous voltage, N_s is the number of turns in the secondary coil, and dΦ/dt is the derivative of the magnetic flux Φ through one turn of the coil

If the turns of the coil are oriented perpendicularly to the magnetic field lines, the flux is the product of the magnetic flux density *B* and the area *A* through which it cuts. The area is constant, being equal to the cross-sectional area of the transformer core, whereas the magnetic field varies with time according to the excitation of the primary.

Since the same magnetic flux passes through both the primary and secondary coils in an ideal transformer, the instantaneous voltage across the primary winding equals

$$V_P = N_P \frac{d\Phi}{dt}.$$

Taking the ratio of the above two equations gives the same voltage ratio and turns ratio relationship shown above, that is,

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} = a$$

The changing magnetic field induces an emf across each winding. The primary emf, acting as it does in opposition to the primary voltage, is sometimes termed the counter emf. This is in accordance with Lenz's law, which states that induction of emf always opposes development of any such change in magnetic field.

As still lossless and perfectly-coupled, the transformer still behaves as described above in the ideal transformer.

Polarity:

Instrument transformer, with polarity dot and X1 markings on LV side terminal

The relationships of the instantaneous polarity at each of the terminals of the windings of a transformer depend on the direction the windings are wound around the core. Identically wound windings produce the same polarity of voltage at the corresponding terminals. This relationship is usually denoted by the dot convention in transformer circuit diagrams, nameplates, and on terminal markings, which marks the terminals having an in-phase relationship.

5.2.6 Real Transformer

The ideal transformer model neglects the following basic linear aspects in real transformers:

- Core losses, collectively called magnetizing current losses, consist of Hysteresis losses due to nonlinear application of the voltage applied in the transformer core, and
- Eddy current losses due to joule heating in the core that are proportional to the square of the transformer's applied voltage.
- Whereas windings in the ideal model have no impedance, the windings in a real transformer have finite non-zero impedances in the form of:
- Joule losses due to resistance in the primary and secondary windings
- Leakage flux that escapes from the core and passes through one winding only resulting in primary and secondary reactive impedance.

If a voltage is applied across the primary terminals of a real transformer while the secondary winding is open without load, the real transformer must be viewed as a simple inductor with an impedance Z

$$Z_P = j\omega L_P \qquad I_{\mathrm{P}} = V_{\mathrm{P}}/Z_{\mathrm{P}}$$